B.Sc. Sem- I

Mathematics Hons.

Paper — Mat C-2

Group- A (Differential Calculus)

Book:

Differential Calculus

— By Lalji Prasad.

Successive Differentiation.

Notation:

Given function $y = f(x)$

first derivative $\frac{dy}{dx}$ by y_1

second derivative $\frac{d^2y}{dx^2}$ by y_2.

third derivative $\frac{d^3y}{dx^3}$ by y_3.

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n^{th} derivative $\frac{d^ny}{dx^n}$ by y_n.

n^{th} differential Coefficients of some standard functions.

1) n^{th} differential of x^m.

Let $y = x^m$.

Then $y_1 = \frac{dy}{dx} = mx^{m-1}$

$y_2 = \frac{d^2y}{dx^2} = m(m-1)x^{m-2}$

$y_3 = \frac{d^3y}{dx^3} = m(m-1)(m-2)x^{m-3}$

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Let us suppose that

$y_n = m(m-1)(m-2)(m-3)\ldots(m-n+1)x^{m-n}$ —— (1)

Differentiating again, we get

$y_{n+1} = m(m-1)(m-2)(m-3)\ldots(m-n+1)(m-n)x^{m-n-1}$

which is of the same form as (1) with n replaced by $(n+1)$.

Since eq^n (1) is true for $n = 1, 2, 3$

Therefore, by the method of induction (1) is true $\forall\, n$.

2) nth differential coefficient of $\frac{1}{ax+b}$.

Let $y = \frac{1}{ax+b} = (ax+b)^{-1}$

$\therefore \quad y_1 = (-1)(ax+b)^{-2}\, a$

$y_2 = (-1)(-2)(ax+b)^{-3}\, a^2.$

$y_3 = (-1)(-2)(-3)(ax+b)^{-4}\, a^3.$

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Let us suppose that

$y_n = (-1)(-2)(-3)\ldots\ldots(-n)(ax+b)^{-(n+1)}\, a^n. \quad \text{—— (1)}$

$= (-1)^n a^n n!\, (ax+b)^{-(n+1)}$

$= \frac{(-1)^n a^n n!}{(ax+b)^{n+1}} \quad \text{—— (2)}$

Differentiating (1) again w.r.t. x, we get

$y_{n+1} = (-1)(-2)(-3)\ldots(-n)\{-(n+1)\}(ax+b)^{-(n+2)}\, a^{n+1}$

$= \frac{(-1)^{n+1}(n+1)!\, a^{n+1}}{(ax+b)^{n+2}}.$

which is of the same form as (2) with n replaced by $n+1$.

Since (2) is true for $n = 1, 2, 3.$

$\therefore$ By the method of induction (2) is true for all n.

8). Find the nth differial differential Coefficient of e^{mx}, $\log(ax+b)$.

5) nth differential Coefficient of $\sin(ax+b)$

let $y = \sin(ax+b)$

$$y_1 = a\cos(ax+b) = a\sin\left(ax+b+\frac{\pi}{2}\right)$$

$$\because \sin\left(\frac{\pi}{2}+\theta\right) = \cos\theta$$

$$y_2 = a^2\cos\left(ax+b+\frac{\pi}{2}\right)$$

$$= a^2\sin\left(ax+b+\frac{\pi}{2}+\frac{\pi}{2}\right)$$

$$= a^2\sin\left(ax+b+2\cdot\frac{\pi}{2}\right)$$

$$y_3 = a^3\cos\left(ax+b+2\cdot\frac{\pi}{2}\right)$$

$$= a^3\sin\left(ax+b+2\cdot\frac{\pi}{2}+\frac{\pi}{2}\right)$$

$$= a^3\sin\left(ax+b+3\frac{\pi}{2}\right).$$

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let us suppose that

$$y_n = a^n\sin\left(ax+b+n\frac{\pi}{2}\right) \quad \text{---} (1)$$

Differentiating ① again w.r.t. x, we get

$$y_{n+1} = a^{n+1} \cos\left(ax+b+n\frac{\pi}{2}\right).$$

$$= a^{n+1} \sin\left(ax+b+n\frac{\pi}{2}+\frac{\pi}{2}\right).$$

$$= a^{n+1} \sin\left\{ax+b+(n+1)\frac{\pi}{2}\right\}$$

which is of the same form as ① with n replaced by $(n+1)$.

Since ① is true for $n=1,2,3.....$
$\therefore$ by the method of induction it is true for all n.

9). Find the nth differential coefficient of $\cos(ax+b)$, $e^{ax}\sin(bx+c)$.